

# Shear viscosity of nuclear matter

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**Abstract** This paper reports my recent study<sup>[1]</sup> on the shear viscosity of neutron-rich nuclear matter from a relaxation time approach. An isospin- and momentum-dependent interaction is used in the study. Dependence of density, temperature, and isospin asymmetry of nuclear matter on its shear viscosity have been discussed. Similar to the symmetry energy, the symmetry shear viscosity is defined and its density and temperature dependence are studied.

**Key words** Shear viscosity, Relaxation time, Nuclear matter, Isospin asymmetry

## 1 Introduction

One of the major problems in nuclear physics is to understand the properties of nuclear matter under extreme conditions. This is related to the basic knowledge of the in-medium nucleon-nucleon ( $NN$ ) interaction which in the present stage can still hardly be obtained from the ab initio theory of the strong interaction, i.e., Quantum chromodynamics. Our knowledge on the in-medium  $NN$  interaction today is mainly developed along two lines. In the first line, one starts from the bare  $NN$  interaction, which has been fitted very well from  $NN$  scattering data, together with phenomenological three-body interactions, so that the in-medium  $NN$  interaction and the properties of nuclear matter can be obtained through many-body theories. In the second line, the starting point is an effective in-medium  $NN$  interaction or Lagrangian, with the parameters fitted to the empirical nuclear matter properties obtained usually through mean-field approximations.

Ten years ago, an isospin- and momentum-dependent mean-field potential (hereafter 'MDI') was constructed to study the dynamics (especially the isospin effects) in intermediate-energy heavy-ion collisions together with an isospin-dependent Boltzmann-Uehling-Uhlenbeck (IBUU) transport model<sup>[2]</sup>. In addition to the good

description of the empirical nuclear equation of states, the momentum dependence of this mean-field potential reproduces pretty good the optical potential extracted by Hama *S et al.* from elastic proton scattering data<sup>[3]</sup>. The studies using this interaction have constrained the nuclear symmetry energy at both subsaturation and suprasaturation densities<sup>[4-6]</sup>. In addition to the dynamics of heavy-ion collisions, the MDI model has also been used in the study of thermodynamical properties of nuclear matter<sup>[7,8]</sup>. It was recently found that the isospin- and momentum-dependent potential can be derived from an effective interaction with a density-dependent two-body interaction and a Yukawa-type finite-range interaction using Hartree-Fock calculation<sup>[9]</sup>. The MDI model thus serves as a useful effective in-medium interaction.

In the past few years, the shear viscosity of the quark-gluon plasma (QGP) formed in relativistic heavy-ion collisions has attracted special attentions. From the study using a viscous hydrodynamical model<sup>[10]</sup>, it was found that the strong-interacting QGP behaves like a nearly ideal fluid, i.e., its specific shear viscosity is only a little larger than the Kovtun-Son-Starinets boundary<sup>[11]</sup>. Up to now large efforts have been devoted to study the shear viscosity of QGP<sup>[12-15]</sup> and hadron resonance gas<sup>[16-19]</sup> formed in relativistic heavy-ion collisions, while there are only a

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few studies on the shear viscosity of nuclear matter formed in intermediate-energy heavy-ion collisions<sup>[20-24]</sup>. Even few studies are related to the isospin effects on the shear viscosity of nuclear matter<sup>[25]</sup>. In this paper I will discuss my recent study<sup>[1]</sup> on the shear viscosity of nuclear matter using the MDI model mentioned above from a relaxation time approach, which gives an intuitive picture how the shear viscosity changes with the density, temperature, and isospin asymmetry of nuclear matter.

## 2 Shear viscosity from a relaxation time approach

The system concerned here is an isospin asymmetric nuclear matter with uniform neutron and proton density  $\rho_n$  and  $\rho_p$ , respectively, and the nucleons are thermalized with temperature  $T$ . The flow field  $\vec{u}$  is static in the  $z$  direction and its magnitude is linear in the coordinate  $x$ , i.e.,  $u_z = cx$  and  $u_x = u_y = 0$ . In the rest frame nucleons move with the flow field and follow Fermi-Dirac distribution  $n^*$  in the equilibrium state. In the lab frame the equilibrium distribution is a simple boost by the flow field compared with that in the rest frame, denoted as  $n^0$ . Due to NN collisions, the real distribution may be slightly away from the equilibrium distribution and is denoted as  $n$ , and the deviation from the equilibrium distribution  $\delta n = n^0 - n$  is much smaller than  $n^0$ .

The shear force between flow layers per unit area by definition can be written as

$$\frac{F}{A} = \sum_{\tau} \langle (p_z - mu_z) \rho_{\tau} v_x \rangle. \quad (1)$$

In the above,  $\tau = n$  or  $p$  denotes the isospin degree of freedom,  $\rho_{\tau} v_x$  is the number of nucleons moving between layers per unit time per unit area, and  $p_z - mu_z$  is the momentum transfer per nucleon in the  $z$  direction. The nucleon velocity in the  $x$  direction  $v_x$  can be further written as  $v_x = p_x / m_{\tau}^*$ , with  $m_{\tau}^*$  being the effective mass. Using the momentum distribution  $n_{\tau} = n_{\tau}^0 + \delta n_{\tau}$  to calculate the average and taking into account that the equilibrium momentum distribution  $n_{\tau}^0$  is even in  $p_x$ , Eq.(1) can be further written as

$$\frac{F}{A} = \sum_{\tau} d \int (p_z - mu_z) \frac{p_x}{m_{\tau}^*} \delta n_{\tau} \frac{d^3 p}{(2\pi)^3}, \quad (2)$$

where  $d = 2$  is the spin degeneracy.

In the following I will calculate  $\delta n_{\tau}$  by linearizing the isospin-dependent BUU equation as follows:

$$\begin{aligned} & \frac{\partial n_{\tau}(p_1)}{\partial t} + \vec{v} \cdot \nabla_r n_{\tau}(p_1) - \nabla_r U_{\tau} \cdot \nabla_p n_{\tau}(p_1) \\ &= - \left( d - \frac{1}{2} \right) \int \frac{d^3 p_2}{(2\pi)^3} \frac{d^3 p'_1}{(2\pi)^3} \frac{d^3 p'_2}{(2\pi)^3} |T_{\tau, \tau}|^2 \\ & \quad \times [n_{\tau}(p_1) n_{\tau}(p_2) (1 - n_{\tau}(p'_1)) (1 - n_{\tau}(p'_2)) \\ & \quad - n_{\tau}(p'_1) n_{\tau}(p'_2) (1 - n_{\tau}(p_1)) (1 - n_{\tau}(p_2))] \\ & \quad \times (2\pi)^3 \delta^{(3)}(\vec{p}_1 + \vec{p}_2 - \vec{p}'_1 - \vec{p}'_2) \\ & - d \int \frac{d^3 p_2}{(2\pi)^3} \frac{d^3 p'_1}{(2\pi)^3} \frac{d^3 p'_2}{(2\pi)^3} |T_{\tau, -\tau}|^2 \\ & \quad \times [n_{\tau}(p_1) n_{-\tau}(p_2) (1 - n_{\tau}(p'_1)) (1 - n_{-\tau}(p'_2)) \\ & \quad - n_{-\tau}(p'_1) n_{-\tau}(p'_2) (1 - n_{\tau}(p_1)) (1 - n_{-\tau}(p_2))] \\ & \quad \times (2\pi)^3 \delta^{(3)}(\vec{p}_1 + \vec{p}_2 - \vec{p}'_1 - \vec{p}'_2). \end{aligned} \quad (3)$$

In the above,  $U_{\tau}$  is the mean-field potential from the MDI model<sup>[2,4]</sup>,  $T$  is the transition matrix, the degeneracy  $d - 1/2$  takes the double counting of identical nucleon collisions into consideration, and  $1 - n$  is from the Pauli blocking effect. Replacing  $n$  with  $n^0$  in the first-order approximation, the left-hand side can be expressed as

$$\begin{aligned} & \frac{\partial n_{\tau}(p_1)}{\partial t} + \vec{v} \cdot \nabla_r n_{\tau}(p_1) - \nabla_r U_{\tau} \cdot \nabla_p n_{\tau}(p_1) \\ &= \left( - \frac{\partial u_z}{\partial x} \frac{p_z p_x}{p} \frac{dn_{\tau}^0}{dp} \right)_{p=p_1} \end{aligned} \quad (4)$$

by using the properties of  $n^0$ . Keeping only the  $\delta n_{\tau}(p_1)$  term, the right-hand side of Eq.(3) can be expressed as  $\delta n_{\tau}(p_1) / \tau_{\tau}(p_1)$ , where  $\tau_{\tau}(p_1)$  is the relaxation time, i.e., the average time between two collisions for a nucleon with isospin  $\tau$  and momentum  $p_1$ , and it can be written as

$$\frac{1}{\tau_{\tau}(p_1)} = \frac{1}{\tau_{\tau}^{\text{same}}(p_1)} + \frac{1}{\tau_{\tau}^{\text{diff}}(p_1)}, \quad (5)$$

where  $\tau_{\tau}^{\text{same(diff)}}(p_1)$  is the average time for a nucleon with momentum  $p_1$  to collide with other nucleons of same (different) isospin, and they can be calculated respectively from

$$\frac{1}{\tau_{\tau}^{\text{same}}(p_1)} = \left(d - \frac{1}{2}\right) \frac{(2\pi)^2}{(2\pi)^3} \int p_2^2 dp_2 d \cos \theta_{12} d \cos \theta$$

$$\times \frac{d\sigma_{\tau,\tau}}{d\Omega} \left| \frac{\vec{p}_1}{m_{\tau}^*(p_1)} - \frac{\vec{p}_2}{m_{\tau}^*(p_2)} \right|$$

$$\times [n_{\tau}^0(p_2) - n_{\tau}^0(p_2)n_{\tau}^0(p_1)$$

$$- n_{\tau}^0(p_2)n_{\tau}^0(p_2') + n_{\tau}^0(p_1')n_{\tau}^0(p_2')],$$
(6)

$$\frac{1}{\tau_{\tau}^{\text{diff}}(p_1)} = d \frac{(2\pi)^2}{(2\pi)^3} \int p_2^2 dp_2 d \cos \theta_{12} d \cos \theta$$

$$\times \frac{d\sigma_{\tau,-\tau}}{d\Omega} \left| \frac{\vec{p}_1}{m_{\tau}^*(p_1)} - \frac{\vec{p}_2}{m_{\tau}^*(p_2)} \right|$$

$$\times [n_{-\tau}^0(p_2) - n_{-\tau}^0(p_2)n_{\tau}^0(p_1')$$

$$- n_{-\tau}^0(p_2)n_{\tau}^0(p_2') + n_{\tau}^0(p_1')n_{-\tau}^0(p_2')].$$
(7)

In the above  $\theta_{12}$  is the angel between  $\vec{p}_1$  and  $\vec{p}_2$ , and  $\theta$  is the scattering angel between the total momentum and the relative momentum of the final state. In free space the pp and np scattering cross sections are isotropic and they can be respectively parameterized as<sup>[26]</sup>

$$\sigma_{pp(nn)} = 13.73 - 15.04/v + 8.76/v^2 + 68.67v^4, \quad (8)$$

$$\sigma_{np} = -70.67 - 18.18/v + 25.26/v^2 + 113.85v, \quad (9)$$

where the cross sections are in mb and  $v$  is the velocity of the projectile nucleon with respect to the fixed target nucleon. This parametrization describes very well the experimental data for the beam energy from 10 MeV to 1 GeV<sup>[26]</sup>. It is worth to note that in the most probable collision energies the  $np$  scattering cross section is about three times the  $pp$  scattering cross section. In nuclear matter, the in-medium NN scattering cross sections are modified by the in-medium effective mass in the form of<sup>[5]</sup>

$$\sigma_{NN}^{\text{medium}} = \sigma_{NN} \left( \frac{\mu_{NN}^*}{\mu_{NN}} \right)^2, \quad (10)$$

where  $\mu_{NN}$  ( $\mu_{NN}^*$ ) is the free-space (in-medium) reduced mass of colliding nucleons.

Once the relaxation time  $\tau_{\tau}(p)$  is known,  $\delta n_{\tau}(p)$  can be calculated from

$$\delta n_{\tau}(p) = \tau_{\tau}(p) \frac{\partial u_{\tau}}{\partial x} \frac{p_z p_x}{p} \frac{dn_{\tau}^0}{dp}. \quad (11)$$

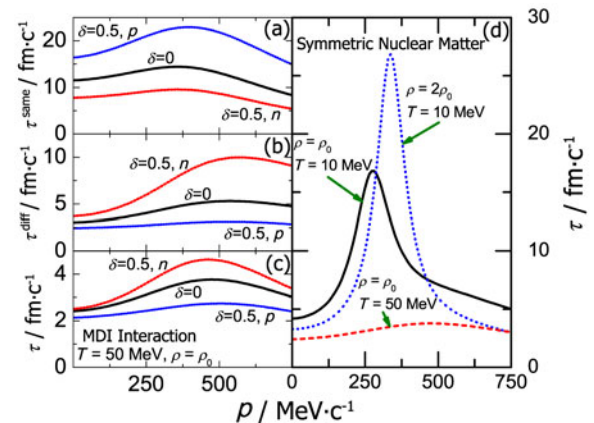
Using the definition  $F/A = -\eta(\partial u_{\tau}/\partial x)$ , the shear viscosity can be calculated from Eqs.(2) and (11) in terms of the local momentum distribution  $n_{\tau}^*$  as

$$\eta = \sum_{\tau} -d \int \tau_{\tau}(p) \frac{p_z^2 p_x^2}{p m_{\tau}^*} \frac{dn_{\tau}^*}{dp} \frac{d^3 p}{(2\pi)^3} \quad (12)$$

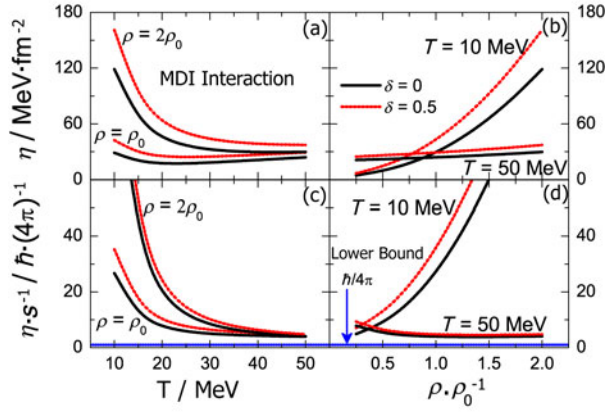
by setting the magnitude of the velocity field to be infinitely small. Note that from Eq.(12) the shear viscosity is related to the local momentum distribution near the Fermi surface.

### 3 Results and discussion

Figure 1 displays the density, temperature, and isospin dependence of the relaxation time. In neutron-rich nuclear matter,  $\tau_n^{\text{diff}}$  is larger while  $\tau_n^{\text{same}}$  is smaller compared to that in symmetric nuclear matter as a result of less frequent  $np$  collisions and more frequent  $nn$  collisions. For the similar reason,  $\tau_p^{\text{same}}$  is larger while  $\tau_p^{\text{diff}}$  is smaller compared to that in symmetric nuclear matter. From Eq.(5), the total relaxation time is determined by  $\tau^{\text{diff}}$  which is always smaller than  $\tau^{\text{same}}$  due to the larger  $np$  cross section than  $pp(nn)$  cross section in the most probable collision energies. Thus, neutrons have a larger relaxation time than protons in neutron-rich nuclear matter. It is seen in Panel (d) that the relaxation time decreases with increasing temperature due to more frequent collisions at higher temperatures. In addition, at lower temperatures the relaxation time peaks around the Fermi momentum, indicating a strong Pauli blocking effect for nucleons near the Fermi surface.



**Fig.1** (Color online) Panel (a), (b), (c): Relaxation time for neutrons and protons as a function of nucleon momentum in symmetric ( $\delta = 0$ ) and asymmetric ( $\delta = 0.5$ ) nuclear matter at saturation density and temperature  $T = 50$  MeV; Panel (d): Relaxation time as a function of nucleon momentum in symmetric nuclear matter at different densities and temperatures.



**Fig.2** (Color online) Density and temperature dependence of the shear viscosity ((a), (b)) and specific shear viscosity ((c), (d)) for symmetric ( $\delta=0$ ) and asymmetric ( $\delta=0.5$ ) nuclear matter.

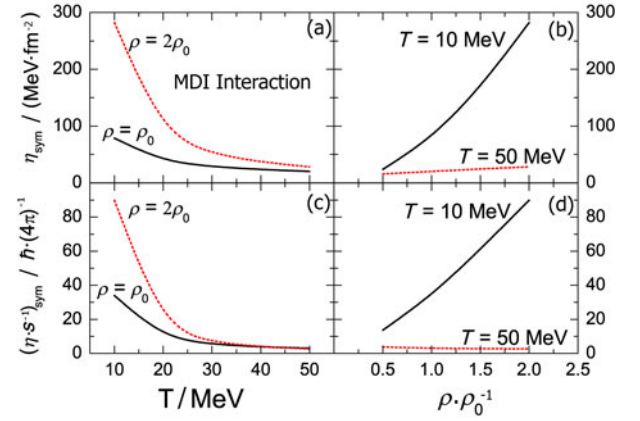
Results of the shear viscosity  $\eta$  and specific shear viscosity  $\eta/s$ , where  $s$  is the entropy density, are shown in Fig.2. The temperature dependence of the shear viscosity is similar to that in Ref.[20] at different densities, while  $\eta$  increases with increasing density especially at lower temperatures due to the strong Pauli blocking effect. The specific shear viscosity decreases with increasing temperature, and it is similar in both magnitude and trend to those obtained from BUU calculations using the Green-Kubo formula<sup>[24]</sup>. It is interesting to see that at higher temperatures the specific shear viscosity is about 4 ~ 5 times the lower limit from Ads/CFT calculation<sup>[11]</sup>, and this is already close to that of QGP extracted from the study using a viscous hydrodynamical model<sup>[10]</sup>. At lower temperatures the specific viscosity increases with increasing density due to the Pauli blocking effect, while at higher temperatures the dependence on the density is rather weak.

Due to the sharper momentum distribution of neutrons compared to that of protons in neutron-rich nuclear matter, the total shear viscosity is dominated by neutrons which have a longer relaxation time in asymmetric nuclear matter compared to that in symmetric nuclear matter. This is confirmed in Fig.2 that both the shear viscosity and the specific shear viscosity are larger in neutron-rich nuclear matter. In addition, it was seen<sup>[1]</sup> that both the shear viscosity and specific shear viscosity satisfy the parabolic approximation with respect to the isospin asymmetry, i.e.,

$$\eta(\rho, T, \delta) \approx \eta(\rho, T, \delta=0) + \eta_{\text{sym}}(\rho, T)\delta^2, \quad (13)$$

$$\left(\frac{\eta}{s}\right)(\rho, T, \delta) \approx \left(\frac{\eta}{s}\right)(\rho, T, \delta=0) + \left(\frac{\eta}{s}\right)_{\text{sym}}(\rho, T)\delta^2. \quad (14)$$

Similar to the symmetry energy, the second-order coefficient can thus be defined as the symmetry shear viscosity or the symmetry specific shear viscosity. The density and temperature dependence of them are shown in Fig.3. It is seen that both the symmetry shear viscosity and symmetry specific shear viscosity decrease with increasing temperature. At lower temperatures, both of them increase with increasing density. At higher temperatures, the density dependence is rather weak.  $\eta_{\text{sym}}$  and  $(\eta/s)_{\text{sym}}$  are important quantities in understanding transport properties of neutron-rich nuclear matter, and they deserve further studies in the future.



**Fig.3** (Color online) Density and temperature dependence of symmetry shear viscosity ((a) and (b)) and symmetry specific shear viscosity ((c) and (d)).

## 4 Conclusion

Using a relaxation time approach, I studied the shear viscosity and specific shear viscosity of hot neutron-rich nuclear matter as that formed in intermediate-energy heavy-ion collisions by using an isospin- and momentum-dependent interaction. It is found that the specific shear viscosity decreases with increasing temperature, and it increases with increasing density at lower temperatures due to the strong Pauli blocking effect. Furthermore, both the shear viscosity and specific shear viscosity are found to increase with increasing isospin asymmetry of nuclear matter and roughly satisfy the parabolic approximation. The second-order coefficient in the

expansion of the isospin asymmetry, which is defined as the symmetry shear viscosity or the symmetry specific shear viscosity, has also been studied.

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